***Discrete Math*. Handwriting Assignment #3**

1. a) Is the number 0 in ∅?   
   b) Is ∅ = {∅}?  
   c) Is ∅ ∈ {∅}?   
   **Solution**  
   a) No. The empty set contains no elements.   
   b) No. The set {∅} has one element, namely ∅, whereas ∅ contains no elements.  
   c) Yes. The set {∅} has one element, namely ∅. That is, ∅ ∈ {∅}.
2. Let A and B be two sets. Prove that (A − B) ∩ (A ∩ B) = ∅.  
   **Solution**  
    Suppose not. Then there is an x ∈ (A − B) ∩ (A ∩ B). This implies that x ∈ A − B, x ∈ A, and x ∈ B. But x ∈ A − B implies that x ∈ A and x ∉ B. So we have x ∈ B and x ∉ B (This is a contradiction!) Hence, (A−B)∩(A∩B) = ∅
3. Let A and B be two sets. Show that if A ⊆ B then A ∩ Bc = ∅.  
   **Solution**  
   Suppose that A ∩ Bc ≠ ∅. Then there is an x ∈ A ∩ Bc . This implies that x ∈ A and x ∈ Bc . Thus, x ∈ A and x ∉ B. Since A ⊆ B and x ∈ A, we have x ∈ B. It follows that x ∈ B and x ∉ B which is impossible
4. Show that if n is an odd integer then ⎡n/2⎤ = (n+1)/2 .  
   **Solution**  
   Let n be an odd integer. Then there is an integer k such that n = 2k − 1. Hence, n/2 = k – 1/2  
   
5. Certain automobile license plates consist of a sequence of three letters followed by three digits.  
   a) If letters cannot be repeated but digits can, how many possible license plates are there?  
   b) If no letters and no digits are repeated, how many license plates are possible?  
   **Solution** a) 26x25x24x10x10x10  
    b) 26x25x24x10x9x8
6. In how many ways can three distinct numbers be chosen from the set {1, 2, . . . , 100} such that their sum is even?  
   **Solution**   
   There are 50 even and 50 odd numbers. So to have an even sum  
   we can either choose three even numbers, or choose two odd numbers and 1 even numbers. Therefore C(50,3)+C(50,2)C(50,1)
7. a) Give a recursive definition of a function named repeat that takes three parameters. The first parameter is a function from integers to integers, the second parameter is a natural number, and the third is an integer. The output of repeat(f, n, x) should be the result of applying f to x, then applying f again to the result, and so on, n times.  
   For example, suppose add1 (n) = n + 1, then   
    repeat(add1 , 3, 5) = add1 (add1 (add1 (5))) = 8  
    repeat(add1 , 0, 5) = 5  
   b) Prove that for any f ∈ Z → Z, m ∈ N, n ∈ B, x ∈ Z,  
    repeat(f, n + m, x) = repeat(f, m, repeat(f, n, x)).

